Transmission of an Optical Wave Beam Through a System of Two Aperture Stops

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Abstract—The beam-mode expansion method used in the discussions of the diffraction of a Gaussian wave beam through an aperture is applied to a system of two circular or square aperture stops, and the analytical expressions of the power transmission and conversion coefficients of a fundamental mode through the system are obtained.

By using these expressions, the optimum incidence conditions that maximize the power transmission coefficient of the fundamental mode can be known. These conditions coincide formally with those obtained by Kogelnik and Yariv for an incident wave having a prolate spheroidal-wave function distribution.

Both circular and square geometries can be analyzed in the same way.

I. Introduction

ONE of the important problems in optical systems is to know the diffraction effects of a wave beam by an aperture. This problem has been discussed in the literature [1]–[3]. The most direct method [2] gives the distributions of the diffraction field. But the beam-mode expansion method [1], [3] is applicable to the analyses of more complicated systems. Particularly, an optical system that consists of two aperture stops has a practical importance as the noise reduction scheme for laser amplifiers [4]. This system has first been analyzed for the incident wave having a prolate spheroidal-wave function distribution by using the generalized confocal resonator theory [5], [6]. However, practical wave beams that are often used in optical transmission have a Gaussian distribution [7]. The purpose of this paper is, therefore, to investigate the transmission of a Gaussian wave beam through two consecutive apertures. This result is useful to the design of noise filters.

In Section II, the analytical expressions of the mode transmission and the mode conversion coefficients for an optical wave beam through a circular or square aperture are given. In Section III, the beam-mode expansion method is applied to the system of two aperture stops when the incident wave beam is a fundamental mode. In Section IV, the optimum conditions which maximize the power transmission coefficient of the fundamental mode are obtained, and the input power loss caused by the blocking area of the first aperture is given in the case of the optimum incidence.

Numerical results are shown for circular geometries.

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II. Mode Transmission and Mode Conversion Coefficients through an Aperture for an Optical Wave Beam

The optical wave beam which has its smallest spot size w_s at $z = -z_s$ is given by [7]

$$\psi_{mn}(r,\theta,z) = \left(\frac{2n!}{\pi\epsilon_m(n+m)!}\right)^{1/2} \cdot \exp\left[-jk(z+z_s)\right]\eta(\eta r)^m L_n{}^m(\eta^2 r^2) \cdot \exp\left[-\tfrac{1}{2}\eta^2\sigma^2 r^2 + j(2n+m+1)\tan^{-1}\xi\right] \cdot \cos(m\theta) \tag{1}$$

where k is the wavenumber of the field which is related to the wavelength λ by $k = 2\pi/\lambda$, and

$$\epsilon_m = \begin{cases} 2, & \text{for } m = 0 \\ 1, & \text{for } m \neq 0 \end{cases} \tag{2}$$

$$\xi = \frac{2(z+z_s)}{kw_s^2} \qquad \eta = \frac{\sqrt{2}}{w_s(1+\xi^2)^{1/2}} \tag{3}$$

$$\sigma^2 = 1 + j\xi. \tag{4}$$

The generalized Laguerre polynomial $L_n{}^m(X)$ is given by

$$L_n{}^m(X) = \sum_{i=0}^{n} \binom{n+m}{n-i} \frac{(-X)^i}{i!} \tag{5}$$

where

$$\binom{n+m}{n-i} = {}_{n+m}C_{n-i}$$

is the binomial coefficient.

Let this optical wave beam be normally incident on a circular aperture of radius a which is located at $z = z_0$ as shown in Fig. 1. The propagation axis of the wave beam is assumed to pass through the center of the aperture. The diffraction field in the Fresnel or the Fraunhofer region is obtained by using the Kirchhoff–Huygens diffraction formula. This is a good approximation when the aperture radius a is much larger than the wavelength of the field [8].

This diffraction field is then expanded into a series of beam-mode functions as follows:

$$U_{mn}(r,\theta,z) = \sum_{\bar{m},\bar{n}} C_{mn}{}^{\bar{m}\bar{n}} \psi_{\bar{m}\bar{n}}(r,\theta,z). \tag{6}$$

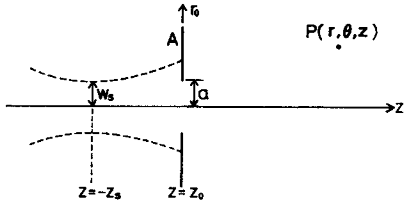

Fig. 1. An incident optical wave beam and an aperture A.

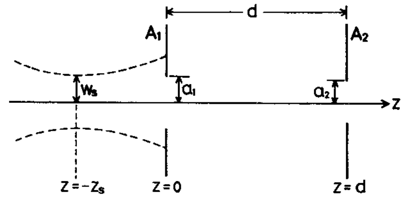

Fig. 2. A system of two aperture stops and an incident optical wave beam.

The expansion coefficients $\{C_{mn}{}^{\bar{m}\bar{n}}\}$ are given by [3]

$$C_{mn}{}^{\bar{m}\bar{n}} = \left(\frac{n!}{(n+m)!}\right)^{1/2}\left(\frac{\bar{n}!}{(\bar{n}+m)!}\right)^{1/2} \cdot \exp\left[j2(n-\bar{n})\tan^{-1}\xi_0\right] \cdot \sum_{p=0}^{n}\sum_{q=0}^{\bar{n}} \frac{(-1)^{p+q}(p+q+m)!}{p!q!} \cdot \binom{n+m}{n-p}\binom{\bar{n}+m}{\bar{n}-q}\left[1-\exp(-\eta_0{}^2a^2) \cdot \sum_{s=0}^{p+q+m} \frac{1}{s!}(\eta_0{}^2a^2)^s\right] \tag{7}$$

for $m = \bar{m}$, and for $m \neq \bar{m}$

$$C_{mn}{}^{\bar{m}\bar{n}} = 0 \tag{8}$$

where ξ_0 and η_0 represent the values of ξ and η at the position of the aperture.

The beam-mode function $\{\psi_{mn}\}$ are orthonormal, therefore, these coefficients can be regarded as the mode transmission and the mode conversion coefficients [9].

For rectangular geometries, the incident wave beam is given by [10]

$$\phi_{mn}(x,y,z) = \frac{\eta}{(\pi 2^{m+n}m!n!)^{1/2}} \cdot \exp\left[-jk(z+z_s)\right]H_m(\eta x)H_n(\eta y) \cdot \exp\left[-\tfrac{1}{2}\eta^2\sigma^2(x^2+y^2) + j(m+n+1)\tan^{-1}\xi\right] \tag{9}$$

and the mode transmission and the mode conversion coefficients are, by changing the results shown in [3] into a more convenient and more generalized form [11], given as follows:

$$C_{mn}{}^{\bar{m}\bar{n}} = C_m{}^{\bar{m}}C_n{}^{\bar{n}} \tag{10}$$

$$C_t{}^{\bar{t}} = \frac{1+(-1)^{t+\bar{t}}}{2(2^{t+\bar{t}+1}t!\bar{t}!)^{1/2}} \cdot \exp\left[j(t-\bar{t})\tan^{-1}\xi_0\right] \cdot \sum_{p=0}^{[t/2]}\sum_{q=0}^{[\bar{t}/2]} \frac{(-1)^{p+q}t!\bar{t}!}{p!q!(t-2p)!(\bar{t}-2q)!} \cdot (2h-1)!!(\sqrt{2})^{2h+1}\left[2\Phi(\sqrt{2}\eta_0 a) - (2-\epsilon_h)\left(\frac{2}{\pi}\right)^{1/2}\exp(-\eta_0{}^2a^2) \cdot \sum_{s=0}^{h-1}\frac{1}{(2s+1)!!}(\sqrt{2}\eta_0 a)^{2s+1}\right] \tag{11}$$

where

$$h = \tfrac{1}{2}(t+\bar{t}-2p-2q) \tag{12}$$

$$(2h-1)!! = (2h-1)(2h-3)\cdots 3\cdot 1, \qquad (-1)!! = 1 \tag{13}$$

and $2a$ is the length of the side of the aperture.

The error function $\Phi(X)$ and the Hermite function $H_t(X)$ are given by

$$\Phi(X) = \frac{1}{(2\pi)^{1/2}}\int_0^X \exp(-Y^2)\,dY \tag{14}$$

$$H_t(X) = (-1)^t\exp(X^2)\frac{d^t}{dX^t}\exp(-X^2). \tag{15}$$

III. Power Transmission Coefficient of a Fundamental Mode through a System of Two Aperture Stops

In this section, the case of a fundamental mode incidence is considered. Let the fundamental mode ψ_{00} be incident on a system of two aperture stops as shown in Fig. 2. The apertures A_1 and A_2 are separated by distance d, which is much larger than the radii a_1 and a_2 of the apertures A_1 and A_2, respectively.

The power of the fundamental mode contained in the output field, or in other words, the power transmission coefficient of this mode through the system, is given by

$$\tau = \left|\sum_{n=0}^{\infty} C_{00}{}^{(1)0n}C_{0n}{}^{(2)00}\right|^2 \tag{16}$$

where

$$C_{00}{}^{(i)0n} = C_{0n}{}^{(i)00}\exp\left[-j4n\tan^{-1}\xi_{i0}\right] = \exp\left[-j2n\tan^{-1}\xi_{i0}\right]\sum_{p=0}^{n}(-1)^p\binom{n}{n-p} \cdot \left[1-\exp(-\eta_{i0}{}^2a_i{}^2)\sum_{s=0}^{p}\frac{1}{s!}(\eta_{i0}{}^2a_i{}^2)^s\right] \quad (i=1,2) \tag{17}$$

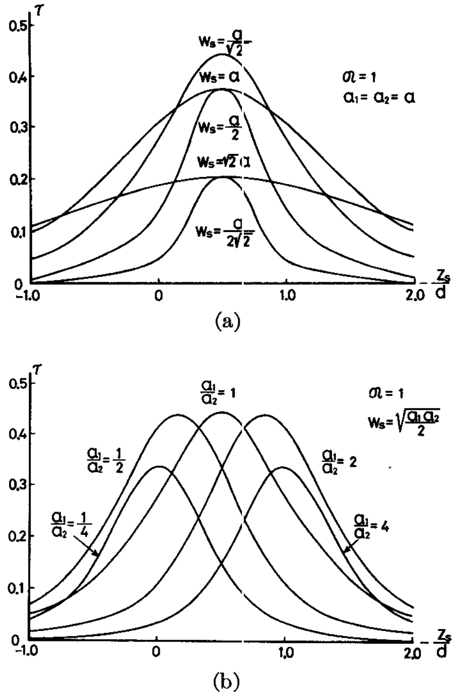

Fig. 3. The power transmission coefficient τ of the fundamental mode as a function of the position of the beam waist $(-z_s)$ which is normalized with respect to the separation d of the apertures. The acceptance factor $\mathfrak{A}$ is equal to unity. (a) The aperture radii a_1 and a_2 are equal to a. (b) The smallest spot size w_s is equal to $(a_1a_2/2)^{1/2}$.

$$\xi_{10} = \frac{2z_s}{kw_s^2} \qquad \eta_{10} = \frac{\sqrt{2}}{w_s(1+\xi_{10}^2)^{1/2}} \tag{18}$$

$$\xi_{20} = \frac{2(z_s+d)}{kw_s^2} \qquad \eta_{20} = \frac{\sqrt{2}}{w_s(1+\xi_{20}^2)^{1/2}}. \tag{19}$$

Introducing the following parameters

$$\mathfrak{A} = \frac{\pi^2 a_1^2 a_2^2}{\lambda^2 d^2} \tag{20}$$

$$\beta_s = \frac{(a_1a_2)^{1/2}}{w_s} \tag{21}$$

we can rewrite (18) and (19) as follows:

$$\eta_{10} = \beta_s\left(\frac{2}{a_1a_2(1+\xi_{10}^2)}\right)^{1/2} \tag{22}$$

$$\xi_{20} = \xi_{10} + \frac{\beta_s^2}{(\mathfrak{A})^{1/2}} \qquad \eta_{20} = \beta_s\left(\frac{2}{a_1a_2(1+\xi_{20}^2)}\right)^{1/2}. \tag{23}$$

The parameter $\mathfrak{A}$ is defined as the acceptance factor [6].

The power transmission coefficient τ is divided into two parts, that is, it can be represented as the sum of τ_d and τ_h. τ_d represents the fraction of the power of the fundamental mode that passes directly through the optical system without being changed into higher modes by the apertures A_1 and A_2, and is given by

$$\tau_d = \{C_{00}{}^{(1)00}C_{00}{}^{(2)00}\}^2$$
$$= \{1-\exp(-\eta_{10}^2a_1^2)\}^2\{1-\exp(-\eta_{20}^2a_2^2)\}^2. \tag{24}$$

The meaning of $\tau_h = \tau - \tau_d$ can be seen from (16) and (24); τ_h is the contribution due to the mode conversion by the apertures. In calculating τ_h, we must take into consideration the phase term in (17).

The power of the higher modes in the output field, which originated in transmitting the optical system, is given by

$$P_h = \sum_{\bar{n}=1}^{\infty} P_{h\bar{n}} \tag{25}$$

$$P_{h\bar{n}} = \left|\sum_{n=0}^{\infty} C_{00}{}^{(1)0n}C_{0n}{}^{(2)0\bar{n}}\right|^2. \tag{26}$$

The power transmission coefficient τ changes according to the incident conditions. Fig. 3(a) and (b) shows two numerical examples where the acceptance factor $\mathfrak{A} = 1$. In this paper, only the effects of the first three higher modes are considered for numerical computations.

Fig. 3(a) shows the coefficient τ when w_s and z_s of the incident beam are varied, while a_1 and a_2 are kept constant and equal to a. In this case, τ takes its maximum values for $(-z_s)/d = 0.5$ independently of w_s. This means that the position of the beam waist of the incident wave beam is in the middle between two apertures. There are two w_s that give the same maximum value of τ, except when w_s is equal to $a/\sqrt{2}$. But τ decreases from the maximum value more rapidly for the smaller value of them than for the larger one, depending upon z_s.

Fig. 3(b) shows the coefficient τ when a_1/a_2 and z_s are varied, while w_s is kept constant. From this figure, it can be seen that as the ratio a_1/a_2 increases, the position of the beam waist $(-z_s)$ at which τ takes its maximum value approaches the position of the smaller aperture.

For rectangular geometries, the power transmission coefficient is given by

$$\tau = \left|\sum_{m,n=0}^{\infty} C_{00}{}^{(1)mn}C_{mn}{}^{(2)00}\right|^2. \tag{27}$$

This coefficient also is represented as the sum of τ_d and τ_h. τ_d is given by

$$\tau_d = \{C_{00}{}^{(1)00}C_{00}{}^{(2)00}\}^2 = \{4\Phi(\sqrt{2}\eta_{10}a_1)\Phi(\sqrt{2}\eta_{20}a_2)\}^4 \tag{28}$$

where $2a_1$ and $2a_2$ are the length of the side of the square apertures A_1 and A_2, respectively. While τ_h is, as in the case of circular geometries, defined by $\tau_h = \tau - \tau_d$. In this case, the acceptance factor $\mathfrak{A}$ is defined by

$$\mathfrak{A} = \frac{16a_1^2a_2^2}{\lambda^2 d^2} \tag{29}$$

accordingly, ξ_{20} is written as follows:

$$\xi_{20} = \xi_{10} + \frac{4\beta_s^2}{\pi(\mathfrak{A})^{1/2}}. \tag{30}$$

The power of higher modes in the output is given by

$$P_h = \sum_{\bar{m},\bar{n}=0}^{\infty}{}' P_{h\bar{m}\bar{n}} \tag{31}$$

$$P_{h\bar{m}\bar{n}} = \left| \sum_{m,n=0}^{\infty} C_{00}{}^{(1)mn} C_{mn}{}^{(2)\bar{m}\bar{n}} \right|^2 \tag{32}$$

where $\sum'$ represents the sum for $\bar{m} \geq 0$ and $\bar{n} \geq 0$ except for $\bar{m} = \bar{n} = 0$.

IV. Conditions for Optimum Transmission of the Fundamental Mode

The condition of incidence to maximize the power transmission coefficient of the fundamental mode is considered here for the system of two aperture stops.

First of all, when the ratio a_1/a_2 and the acceptance factor $\mathfrak{A}$ are given, τ_d is maximized for circular geometries with respect to ξ_{10} and β_s. This means that we obtain the smallest spot size w_s and the position of the beam waist $(-z_s)$ of the incident wave beam to maximize τ_d. These conditions are represented by

$$\frac{\partial \tau_d}{\partial \xi_{10}} = \frac{\partial \tau_d}{\partial \beta_s} = 0. \tag{33}$$

Or from (24)

$$\frac{\xi_{10}}{(1+\xi_{10}^2)^2} \frac{a_1}{a_2} \exp(-\eta_{10}^2 a_1^2)[1 - \exp(-\eta_{20}^2 a_2^2)] + \frac{\xi_{20}}{(1+\xi_{20}^2)^2} \frac{a_2}{a_1} \exp(-\eta_{20}^2 a_2^2)[1 - \exp(-\eta_{10}^2 a_1^2)] = 0 \tag{34}$$

$$\frac{1}{(1+\xi_{10}^2)^2} \frac{a_1}{a^2} \exp(-\eta_{10}^2 a_1^2)[1 - \exp(-\eta_{20}^2 a_2^2)] + \frac{1}{(1+\xi_{20}^2)^2} \frac{a_2}{a_1} \exp(-\eta_{20}^2 a_2^2) \left\{(1+\xi_{20}^2) - 2\xi_{20} \frac{\beta_s^2}{(\mathfrak{A})^{1/2}}\right\} [1 - \exp(-\eta_{10}^2 a_1^2)] = 0. \tag{35}$$

From these, we obtain the following equation:

$$\beta_s^2 = -\frac{1+\xi_{10}^2}{\xi_{10}} (\mathfrak{A})^{1/2}. \tag{36}$$

Therefore, ξ_{10} must be negative.

Substituting (36) into (34) or (35), we also obtain

$$F(\kappa,\mathfrak{A}) = \kappa \exp[-2\kappa(\mathfrak{A})^{1/2}] \left\{1 - \exp\left[-\frac{2}{\kappa}(\mathfrak{A})^{1/2}\right]\right\} - \frac{1}{\kappa} \exp\left(-\frac{2}{\kappa}(\mathfrak{A})^{1/2}\right) \{1 - \exp[-2\kappa(\mathfrak{A})^{1/2}]\} = 0 \tag{37}$$

where

$$\kappa = -\frac{a_1}{a_2}\frac{1}{\xi_{10}} \ (>0). \tag{38}$$

Equation (37) is solved numerically as shown in Fig. 4.

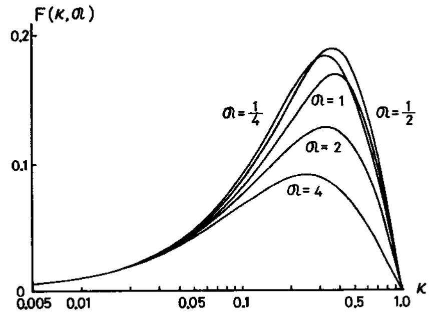

Fig. 4. Numerical values of $F(\kappa, \mathfrak{A})$ defined by (37). For $\kappa > 1$ the values of $F(\kappa, \mathfrak{A})$ are obtained from the fact that $F(\kappa, \mathfrak{A}) = -F(1/\kappa, \mathfrak{A})$.

The solution of this equation which is of any physical significance is given by

$$\kappa = 1. \tag{39}$$

Therefore, we obtain, as the optimum incidence conditions, the following solutions for ξ_{10} and β_s:

$$(\xi_{10})_{\text{opt}} = -\frac{a_1}{a_2} \qquad (\beta_s)_{\text{opt}} = \left(\frac{a_1}{a_2} + \frac{a_2}{a_1}\right)^{1/2} \mathfrak{A}^{1/4} \tag{40}$$

or

$$\left(-\frac{z_s}{d}\right)_{\text{opt}} = \left(1 + \frac{a_2^2}{a_1^2}\right)^{-1}$$

$$(w_s)_{\text{opt}} = (a_1 a_2)^{1/2} \left(\frac{a_1}{a_2} + \frac{a_2}{a_1}\right)^{-1/2} \mathfrak{A}^{-1/4}. \tag{41}$$

From (41) it can be seen that the position of the beam waist $(-z_s)$ is between the apertures A_1 and A_2, that is, $0 \leq (-z_s) \leq d$.

Then th2 maximum value of τ_d is given by

$$(\tau_d)_{\max} = \{1 - \exp[-2(\mathfrak{A})^{1/2}]\}^4. \tag{42}$$

Equations (40) or (41) are obtained to maximize τ_d, but numerical analyses show that these conditions also maximize τ. Therefore, the maximum power transmission coefficient is given by

$$\tau_{\max} = \left| \sum_{n=0}^{\infty} (-1)^n \left\{ \sum_{p=0}^{n} (-1)^p \binom{n}{n-p} \cdot \left[1 - \exp[-2(\mathfrak{A})^{1/2}] \sum_{s=0}^{p} \frac{1}{s!} [2(\mathfrak{A})^{1/2}]^s \right] \right\} \right|^4. \tag{43}$$

In obtaining this expression, we used the following relations:

$$(\xi_{20})_{\text{opt}} = -\frac{1}{(\xi_{10})_{\text{opt}}} \tag{44}$$

$$\tan^{-1}\{(\xi_{20})_{\text{opt}}\} - \tan^{-1}\{(\xi_{10})_{\text{opt}}\} = \frac{\pi}{2}. \tag{45}$$

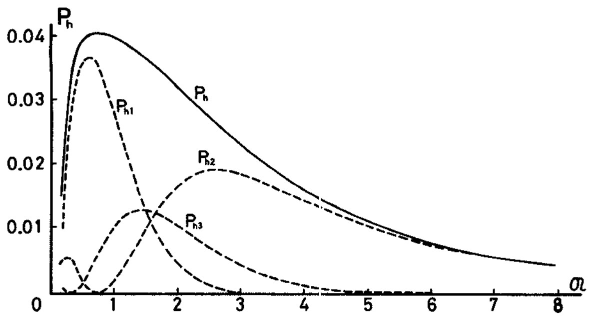

Fig. 5. The power P_h of the higher modes at the output in the case of the optimum incidence. P_h is the sum of $P_{h\bar{n}}$ for $\bar{n} \geq 1$.

The spot size w_1 of the incident wave beam at the position of the aperture A_1 is, in the case of the optimum incidence, given by

$$\frac{(w_1)_{\text{opt}}}{a_1} = \mathfrak{A}^{-1/4} = \left(\frac{\lambda d}{\pi a_1 a_2}\right)^{1/2}. \quad (46)$$

For square apertures, we obtain the following equations:

$$\beta_s^2 = -\frac{\pi}{4}\frac{1+\xi_{10}^2}{\xi_{10}}(\mathfrak{A})^{1/2} \quad (47)$$

$$\kappa^{1/2}\exp\left(-\kappa\frac{\pi}{2}(\mathfrak{A})^{1/2}\right)\Phi[(\kappa\pi)^{1/2}\mathfrak{A}^{1/4}] - \frac{1}{\kappa^{1/2}} \cdot \exp\left(-\frac{\pi}{2}\frac{1}{\kappa}(\mathfrak{A})^{1/2}\right)\Phi\left[\left(\frac{\pi}{\kappa}\right)^{1/2}\mathfrak{A}^{1/4}\right] = 0. \quad (48)$$

These correspond to (36) and (37), respectively. Equation (48) also provides us with the solution

$$\kappa = 1. \quad (49)$$

Therefore, the optimum incidence conditions and the corresponding maximum power transmission coefficient are given by

$$(\xi_{10})_{\text{opt}} = -\frac{a_1}{a_2} \qquad (\beta_s)_{\text{opt}} = \left[\frac{\pi}{4}\left(\frac{a_1}{a_2}+\frac{a_2}{a_1}\right)\right]^{1/2}\mathfrak{A}^{1/4} \quad (50)$$

$$(\tau_d)_{\max} = [2\Phi(\pi^{1/2}\mathfrak{A}^{1/4})]^8 \quad (51)$$

$$\tau_{\max} = \left|\sum_{m,n=0}^{\infty}\frac{\{1+(-1)^m\}^2\{1+(-1)^n\}^2}{16} \cdot \frac{j^{m+n}}{2^{m+n+2}m!n!}D_m^2 D_n^2\right|^2 \quad (52)$$

where

$$D_t = \sum_{p=0}^{[t/2]}\frac{(-1)^p t!}{p!(t-2p)!}(\sqrt{2})^{2g+1}(2g-1)!!\Bigg[2\Phi(\pi^{1/2}\mathfrak{A}^{1/4}) - (2-\epsilon_g)\sqrt{2}\mathfrak{A}^{1/4}\exp\left(-\frac{\pi}{2}(\mathfrak{A})^{1/2}\right) \cdot \sum_{s=0}^{g-1}\frac{1}{(2s+1)!!}[\pi(\mathfrak{A})^{1/2}]^s\Bigg], \qquad (t = m,n) \quad (53)$$

and

$$g = \tfrac{1}{2}(t-2p). \quad (54)$$

The definition of ϵ_g is given by (2). In this case, the spot size of the incident wave beam on the aperture A_1 is

$$\frac{(w_1)_{\text{opt}}}{a_1} = \frac{2}{\pi^{1/2}}\mathfrak{A}^{-1/4} = \left(\frac{\lambda d}{\pi a_1 a_2}\right)^{1/2}. \quad (55)$$

These results expressed by (46) and (55) coincide with those obtained in [6].

The maximum power transmission coefficient of the fundamental mode depends only upon the acceptance factor, independent of the ratio a_1/a_2, as shown by (43) and (52). The power P_h of the higher modes at the output for the case of optimum incidence is shown in Fig. 5.

For reference, the numerical results of $(\tau_d)_{\max}$ and τ_h in the case of the optimum incidence are shown in Table I.

TABLE I
NUMERICAL VALUES OF $(\tau_d)_{\max}$ AND τ_h IN THE CASE OF THE OPTIMUM INCIDENCE

$\mathfrak{a}$	$(\tau_d)_{\max}$	τ_h	$\mathfrak{a}$	$(\tau_d)_{\max}$	τ_h	$\mathfrak{a}$	$(\tau_d)_{\max}$	τ_h
0.1	0.048	-0.029	1.2	0.622	-0.104	3.5	0.908	-0.006
0.2	0.122	-0.060	1.4	0.674	-0.090	4.0	0.929	-0.001
0.3	0.196	-0.084	1.6	0.717	-0.077	4.5	0.944	0.001
0.4	0.265	-0.101	1.8	0.753	-0.064	5.0	0.955	0.003
0.5	0.328	-0.113	2.0	0.784	-0.052	5.5	0.964	0.003
0.6	0.385	-0.119	2.2	0.810	-0.043	6.0	0.971	0.003
0.7	0.436	-0.122	2.4	0.831	-0.033	6.5	0.976	0.003
0.8	0.481	-0.122	2.6	0.850	-0.026	7.0	0.980	0.002
0.9	0.522	-0.119	2.8	0.866	-0.020	7.5	0.983	0.002
1.0	0.559	-0.115	3.0	0.881	-0.015	8.0	0.986	0.001

The power loss caused by the blocking area of the first aperture is given by

$$P_L = \exp(-2\mathfrak{A}^{1/2}) \quad (56)$$

for circular geometries and

$$P_L = 1 - [2\Phi(\pi^{1/2}\mathfrak{A}^{1/4})]^2 \quad (57)$$

for square geometries, both in the case of the optimum incidence. Fig. 6 shows this power loss and $\tau_{\max}$ which is given by (43) or (52).

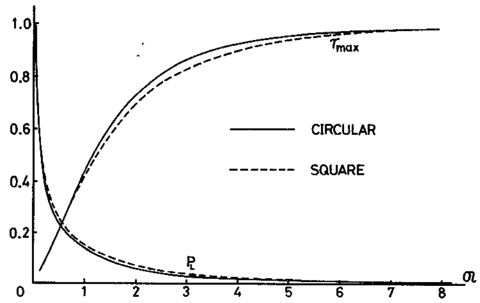

Fig. 6. The power loss P_L caused by the blocking area of the first aperture and $\tau_{\max}$, both in the case of the optimum incidence.

These results will be applied for designing practical noise filters.

V. Conclusions

The power transmission coefficient for an optical wave beam with a Gaussian field distribution through a system of two aperture stops is obtained by using the beam-mode expansion method which has been used to know the diffraction effects of an aperture.

The optimum conditions that maximize the power transmission coefficient of a fundamental beam mode are also obtained. These conditions coincide formally with those given by Kogelnik and Yariv for the incident wave having a prolate spheroidal-wave function distribution. The maximum power transmission coefficient can be represented as a function of only the acceptance factor.

When the noise originated from the spontaneous emission is added to the incident Gaussian wave beam, it is important to obtain the maximum signal-to-noise ratio at the output. This problem could be solved by the method developed here.

The analysis adopted here can be applied to both circular and square geometries, which is one of the characteristics of the beam-mode expansion method.

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Analysis of Electromagnetic-Wave Modes in Anisotropic Slab Waveguide

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***Abstract*—Electromagnetic-wave modes propagating in anisotropic slab waveguide are analyzed theoretically in detail. The propagation conditions are derived under which waves can propagate along the axis of the guide. A two-dimensional three-layered waveguiding structure consisting of an anisotropic dielectric slab coated on, or immersed in, isotropic surrounding substrate materials is considered as a typical configuration of the guide. Field-intensity distributions of the propagating modes and their propagation constants are obtained by numerical computations. Techniques for achieving the mode discrimination and the single-mode operation are given. Some possible applications in integrated optics are suggested.**

I. Introduction

THE electromagnetic-wave modes propagating along a slab waveguide consisting of isotropic materials have been investigated extensively as a typical boundary value problem of electromagnetic-wave theory. For the last few years, this problem has evoked much interest in connection with the development of optical integrated circuits.

On the other hand, the analysis of wave modes in a slab waveguide with anisotropic materials is also of great interest from both the theoretical and practical points of view. To the authors' knowledge, however, little work has been done so far on slab waveguides consisting of anisotropic media, and most of it was restricted to the guide using magnetized gyrotropic ferrites [1].

Recently, Wang *et al.* [2] mentioned the possibility of forming optical devices such as gyrators, optical switches, light modulators, etc., using thin-film waveguide with anisotropic materials as substrates. Nelson and McKenna [3] treated the electromagnetic modes of anisotropic dielectric waveguides at p-n junctions, and Andrews [4] discussed the crystal symmetry effects on nonlinear

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